OCR RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2613/1

Statistics 1

Tuesday **16 JANUARY 2001** Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer all questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

1 The first paragraph of the children's book *Stig of the Dump* contains 107 words. The number of letters per word is summarised in the following table.

Word length (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	2	15	30	23	14	12	6	1	3	1

$$\sum f = 107, \quad \sum fx = 443, \quad \sum fx^2 = 2183.$$

(i) Illustrate the distribution of word lengths by a suitable diagram. [2]

(ii) State the mode, median and mid-range of the data. What feature of the distribution accounts for the different values of these quantities? [4]

(iii) Calculate the mean and standard deviation of the data.

Hence identify the outliers and discuss briefly whether or not they should be excluded from the sample. [6]

A passage of an adult fiction book was analysed in a similar way. The mean number of letters per word was 5.07 and the standard deviation was 2.62.

(iv) Compare the word lengths in the two passages of writing, commenting briefly on the differences. [3]

2 Wendy is an amateur weather forecaster. She classifies the weather on a day as either wet or fine. From past records she suggests that

- if a day is wet then the probability that the next day is wet is 0.5,
- if a day is fine then the probability that the next day is fine is 0.8.

In a particular week, it is wet on Monday.

(i) Draw a probability tree diagram for wet or fine days on Tuesday, Wednesday and Thursday. [4]

(ii) Find the probability that Tuesday, Wednesday and Thursday all have the same weather. [3]

(iii) Find the probability that the weather is wet on Thursday. Given that it is wet on Thursday, find the conditional probability that it was fine on Tuesday. [8]

3 Peter is carrying out a survey for his GCSE *Media Studies* project. He wants to find out the favourite types of television programmes for pupils in his school. In total, there are 1000 pupils on the roll, the numbers in years 7 to 11 being as follows.

Year	7	8	9	10	11
Number of pupils	180	180	200	240	200

Peter wants to take a sample of 50 pupils.

(i) Describe how a systematic sample of 50 pupils may be taken using the sampling frame of all 1000 pupils. [2]

(ii) Name and briefly describe another method of sampling, in which each year is represented proportionately. [3]

Peter carries out his survey with the following results.

Children's programmes	6
Pop programmes	10
Documentaries, current affairs and news	3
Sport	10
Serials and soap operas	13
Comedies	8

(iii) Illustrate the results using a suitable diagram. [2]

(iv) Use the sample information to estimate the number of pupils in the school whose favourite type of programme is *Serials and soap operas*. [2]

Peter chooses 3 pupils at random, from his sample of 50, for a further interview.

(v) In how many ways can Peter choose the 3 pupils? Hence find the probability that all 3 pupils choose either *Pop programmes* or *Sport* as their favourite type of programme. [6]

4 The makers of the drink *Fizzicola* claim that 75% of cola drinkers prefer *Fizzicola* to any other brand of cola.

(i) Assuming that the makers' claim is true, find the probability of precisely three-quarters of a sample of 24 cola drinkers preferring *Fizzicola*. What property must you assume about the nature of the sample? [4]

A rival company suspect that the claim by the makers of *Fizzicola* is exaggerated. They wish to carry out a hypothesis test to examine the claim.

(ii) State suitable null and alternative hypotheses, giving a reason for your choice of the alternative hypothesis. [3]

The rival company take a random sample of 16 cola drinkers, of whom 9 say that they prefer *Fizzicola* to any other brand.

(iii) Carry out the hypothesis test, at the 5% level, stating your conclusion carefully. [3]

The makers of *Fizzicola*, following a promotional campaign, want to test at the 5% level whether or not preference for their brand is now more than 75%. They set up a tasting panel consisting of n randomly chosen cola drinkers.

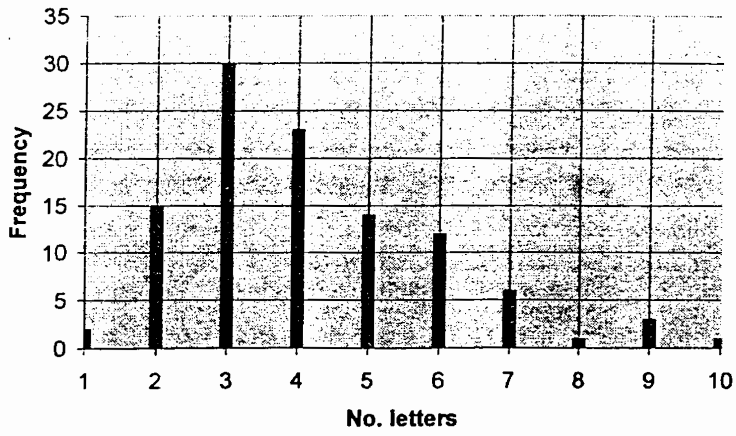
(iv) Show that, when $n = 6$, the critical region for this test is empty. [3]

(v) Find the smallest value of n for which the critical region is not empty. [2]

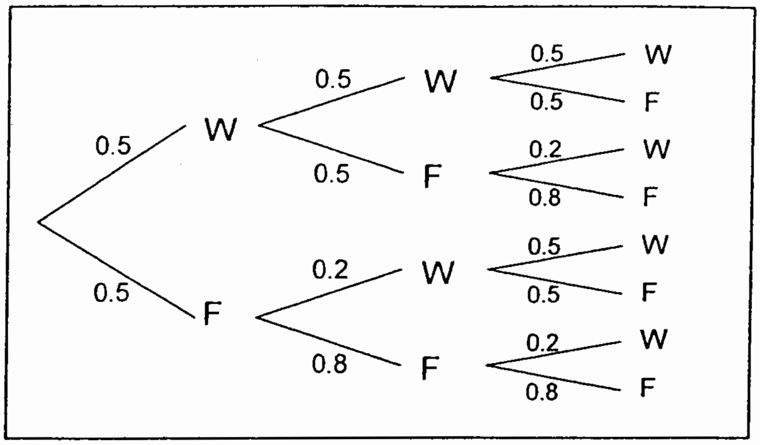
Mark Scheme

January 2001 — 2613 MEI Statistics 1

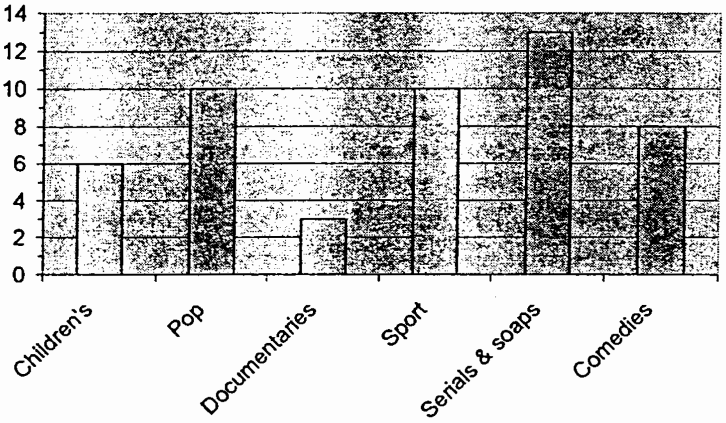
Question 1

(i)		G1 for linear scaled axes, with label on horizontal axis G1 for lines *allow equivalent discrete diagram*	**2**
(ii)	Mode = 3, median = 4, mid-range = 5.5 Distribution's lack of symmetry *or* positive skew *Or* outliers at upper end of distribution	B1, B1, B1 **cao** E1 for explanation	**4**
(iii)	Mean $= \frac{443}{107} = 4.14$ (3 sf) s.d. $= \sqrt{\frac{2183}{107} - 4.14^2} = 1.81$ (3 sf) Mean – 2 s.d. = 4.14 – 2 x 1.81 = 0.52 Mean + 2 s.d. = 4.14 + 2 x 1.81 = 7.76 hence words with 8, 9 or 10 letters are outliers Do not exclude, since occasional long words do occur in such passages	B1 for mean M1 for variance A1 for standard deviation **cao** M1 for use of "2 sd rule" A1 E1	**6**
(iv)	Average number of letters per word greater in the second passage Significantly greater variation in the number of letters per word in the second passage Expect greater proportion of longer words in adult fiction	E1 E1 E1	**3**
			15

Question 2

(i)	 	B1 for overall shape B1 for 1st pair branches B1 for 2nd set branches B1 for 3rd set branches	**4**
(ii)	P(same weather on Tuesday, Wednesday, Thursday) $= 0.5^3 + 0.5 \times 0.8^2 = 0.445$	M1 for 2 triple products, M1 for sum of products A1 **cao**	**3**
(iii)	P(wet Thursday) $= 0.5^3 + 0.5^2 \times 0.2 + 0.5^2 \times 0.2 + 0.5 \times 0.8 \times 0.2$ $= 0.305$ P(fine Tuesday | wet Thursday) $= \dfrac{0.5 \times 0.2 \times 0.5 + 0.5 \times 0.8 \times 0.2}{0.305} = \dfrac{0.13}{0.305}$ $= 0.426$ (3 s.f.) *or* $\dfrac{26}{61}$	M1 for 4 triples A1 for correct triples M1 for sum of 4 products A1 **cao** B1 for numerator B1 for denominator M1 for quotient A1 **cao**	**8**
			15

Question 3

(i)	Choose a random number in the range 1 to 20, say r, then choose the rth, $(r+20)$th, $(r+40)$th, ..., $(r+980)$th pupil from the sampling frame.	E1 for first choice E1 for subsequent choices	**2**
(ii)	Stratified sampling. Number of pupils in each year group in the sample is proportional to number of pupils in each year group in the population. [***or*** list number sampled from each year group: Year 1: 9; year 2: 9; year 3: 10; year 11: 12; year 11: 10]	B1 for method E2 for description	**3**
(iii)		G1 6 bars with labels G1 bars in correct proportions with scale [*Alternatively allow pie chart:* G1 6 sectors with labels G1 sectors in correct proportions]	**2**
(iv)	$13 \times 20 = 260$	M1 A1	**2**
(v)	$^{50}C_3 = 19600$ $\frac{^{20}C_3}{^{50}C_3} = \frac{1140}{19600} = \frac{57}{980} = 0.058$ (2 s.f.) [*or* $\frac{20}{50} \times \frac{19}{49} \times \frac{18}{48} = \frac{57}{980} = 0.058$ (2 s.f.)]	M1 A1 **cao** M1 A1 for $^{20}C_3$ M1 for probability A1 **cao** [*or* M1, M1, M1 A1 **cao**]	**6**
			15

Question 4

(i)	${}^{24}C_{18} \times 0.75^{18} \times 0.25^{6} = 0.185$ (3 s.f.) Sample taken at random	M1 for binomial coefficient M1 for product A1 **cao** E1 for statement	**4**
(ii)	H_0: $p = 0.75$ H_1: $p < 0.75$ Interested to see if proportion is less than 75%	B1 for H_0 B1 for H_1 E1 for explanation *Based on one-tail H_1*	**3**
(iii)	$P(X \le 9) = 0.0796 > 0.05$ (hence accept H_0). There is not enough evidence to reject the hypothesis that the proportion preferring *Fizzicola* is 75%	B1 for probability M1 for comparison A1 for conclusion in words	**3**
(iv)	For $n = 6$: Using tables: $P(X \ge 6) = 1 - 0.8220 = 0.178$ (3 s.f.) *Or* $P(X \ge 6) = 0.75^6 = 0.178$ (3 s.f.) which is greater than 0.05 (hence 6 is not a critical value, hence region is empty)	M1 for calculation of probability A1 **cao** E1	**3**
(v)	Using tables: For $n = 10$: $P(X \ge 10) = 0.0563 > 0.05$ [hence 10 **is not** in the critical region] For $n = 11$: $P(X \ge 11) = 0.0422 < 0.05$ [hence 11 **is** in the critical region] [*or* $0.75n < 0.05 \Rightarrow n > 10.4$] Hence smallest value of n for which critical region is not empty is 11	M1 for either comparison [*or* M1 for inequality] A1 for conclusion	**2**
			15

Examiner's Report

Statistics 1 (2613)

General Comments

The vast majority of students found the paper to be accessible, especially questions 1 and 2, and there was no evidence of students having insufficient time to complete the paper. The question on sampling, new to the specification, was generally not well done, with many students being unclear about the differences between various sampling methods. There was also evidence that some candidates had not fully covered the hypothesis testing section of the specification, since there were many poor attempts at question 4. It was very noticeable that many candidates were far less able to gain explanation marks than calculation marks throughout the paper. Indeed it was common to see candidates making no effort to gain marks on parts which required a thoughtful response.

Comments on Individual Questions

Question 1 (Data analysis; mean and standard deviation, outliers; distribution of word lengths)

Most candidates were able to score well in this question, though there was often confusion about outliers, both in their calculation and interpretation.

(i) Most candidates were able to illustrate the data successfully, but a significant number drew a diagram which implied continuous data. Bar charts, with suitable labelling, were condoned, but they are to be strongly discouraged in favour of the vertical line chart.

(ii) Most candidates were able to identify the mode and the median, but a large proportion did not know how to find the mid-range. A common error was to give an answer of 4.5, which is half of the range. Better candidates could usually identify the positive skewness as being the feature responsible for the differences.

(iii) Calculation of the mean and standard deviation was usually correct, although some candidates re-calculated Σfx and Σfx^2 despite these being given. Others used the statistical functions of the calculator to find the mean and standard deviation, which is of course perfectly acceptable when the original data have been given, as in this case. Some marks were lost due to premature rounding or use of 10, rather than 107, as the divisor.

Not all were able to identify the outliers and many felt that outliers, in general, could only exist above the mean. It was a commonly held view that outliers should *always* be excluded. Candidates should be aware that outliers may be valid members of the data set, in which case they should be included in the analysis, or invalid and so excluded. Identification of outliers should provoke a check of the data's validity.

(iv) Most candidates were able to gain one or two marks here for interpreting the differences in mean and standard deviation. Unfortunately a number did not make it clear whether they were discussing the adult or child data, or did not relate the differences to 'word length'. Others made statements which were not in the context of the data, such as "The adult mean is greater", without referring to either 'word length' or to what the mean measures. There was often confusion between 'number of words' and 'word length'. Full marks demanded a final contextual statement embracing comparisons of central tendency and spread of word lengths.

(i) vertical line chart; (ii) mode = 3, median = 4, mid-range = 5.5;
(iv) mean = 4.14, s.d. = 1.81; outliers: 8, 9, 10; do not exclude; (iv) comments.

Question 2 (Probability; tree diagrams, conditional probability; wet and fine days)

This was found by the majority of candidates to be the easiest question on the paper, with at least 11 marks usually achieved, even by weaker candidates. Many candidates scored full marks.

(i) Most candidates drew a fully correct tree diagram, but a few included 'Monday' and produced a tree with 16 branches, or made errors such as probabilities of 0.5 and 0.8 on each pair of branches.

(ii) This was usually well answered, but a significant number calculated the probability of 3 fine days to be $0.8^3 = 0.512$, despite often drawing a correct tree diagram.

(iii) The probability of Thursday being wet was usually found correctly, although sometimes carelessness resulted in the wrong probability 'triples' being used.

The conditional probability was the only demanding part of the question, and as ever it proved to be a stumbling block for many candidates. There was plenty of evidence of use of the conditional probability formula, but the calculation of P(A and B) was often incorrect. P(A and B) = P(A) × P(B) was often used, which when used in combination with the conditional probability formula, leads to an answer of $P(A|B) = P(A)$. Other candidates calculated P(A and B) correctly but then stopped. However, it was pleasing that many candidates had been well prepared and were able to answer this, as well as the rest of the question, entirely correctly.

(i) tree diagram; (ii) 0.445; (iii) 0.305, 0.426.

Question 3 (Sampling and probability; favourite television programmes of secondary school pupils)

As a new topic for this specification, there was much evidence that candidates were less than sure about which sampling method is appropriate for a particular situation. Indeed, there was much confusion between systematic and stratified sampling.

(i) Many candidates used the definition of *stratified* sampling when describing *systematic* sampling. Even when the correct method was described it was more often than not incomplete. Many answers included "choose every 20th pupil", but very few gave a method for choosing the position of the first pupil, a random integer in the range 1 to 20.

(ii) Most candidates correctly described the sampling method, but often failed to correctly name the procedure as stratified sampling. Full marks were often obtained for a list of the number of pupils sampled in each year group, rather than a description of the process.

(iii) Virtually all candidates were able to draw a satisfactory diagram for categorical data. A bar chart was the most popular and there were several pie charts. A suitably labelled vertical line chart was also condoned.

(iv) Again, the vast majority of candidates were able to estimate the number of pupils whose favourite programme type was Serials and soap operas.

(v) The great majority of students gave the number of choices available to Peter correctly, but very few were able to answer the last part of the question – most candidates attempted a solution based on a binomial distribution, not realising that the calculation should be based on sampling *without* replacement.

(i) description of systematic sampling; (ii) stratified sampling and description.
(iii) bar chart; (iv) 260; (v) 19600, 0.058 (2 s.f.).

Question 4 (Binomial distribution and hypothesis testing; *Fizzicola* brand of cola)

Candidates were often successful with the first three parts of this question, but even the better candidates found parts (iv) and (v) demanding.

(i) Most candidates realised that a binomial distribution was appropriate and correctly found the required probability. However, a disturbing minority thought that all that was required of them was to state that "$\frac{3}{4}$ of 24 is 18"! Less than half realised that the sample must be random. Many mentioned independence, unbiasedness, representativeness or even made some comment on the preferences of the drinkers.

(ii) The hypotheses were given correctly in the majority of cases with the most common errors being sloppy notation. Only a very small minority thought that the test was two tailed. When giving a reason for the choice of alternative hypothesis, a considerable number of candidates described what the alternative hypothesis was rather than saying that it was because there was a suspicion that the proportion was less than 0.75.

(iii) The majority of candidates were able to calculate the correct probability, make a suitable comparison of probabilities and so reject the alternative hypothesis. However, a large number of candidates did not interpret their conclusion in the context of the situation and so lost the final mark. A few attempts at using a "point probability" were seen, which automatically gains no marks at all.

(iv) Candidates found it difficult to write down a convincing justification for this. There was much confusion of inequalities. It was possible to gain credit by comparing $P(X < 5) = 0.8220$ with 0.95, but only if a clear justification was given of why this shows that the critical region is empty. Successful candidates usually calculated $P(X > 6)$ or $P(X = 6)$ and compared this with 0.05. Only the best candidates were able to produce a logical and rigorous justification.

(v) Candidates found this demanding but less than part (iv), since less justification was required. Indeed it was unusual for a candidate to be successful in part (iv), but not so in part (v). An explicit comparison showing that $P(X > 10) > 0.05$ for $n = 10$ but $P(X > 11) < 0.05$ for $n = 11$ was desirable but not required. A few high scoring candidates produced a correct solution using logarithms.

(i) 0.185; (ii) $H_0: p = 0.75$, $H_1: p < 0.75$; (iii) $P(X < 9) = 0.0786 > 0.05$;
(iv) $P(X > 6) = 0.178 > 0.05$; (v) $n = 11$.